Study the Process of Variation of Resistance Coefficients in Non-Uniform Flows

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ABSTRACT
The actual fluid flow in the waterways is in contrast to the ideal fluid flow with the creation of shear stresses and on the one hand the amount of these stresses depends on the physical properties of the fluid and the flow behavior and on the other hand depends on the physical characteristics of the waterway. So, these characteristics affect the environmental resistance against the fluid passing through the environment. The exact measurement of the frictional force of the source of the shear stresses is not feasible, and this has led to many investigators to approximate estimate the resistance level. As a result, investigations were applied to optimize the coefficients in transactions governing the flow of open and close channels. Although the experiments showed that the use of uniform flow resistance coefficients for non-uniform flows are usable, but they all confirmed the necessity of modulating the coefficients for non-uniform flows and here, assuming that the being non-uniform in the passing flow on smooth surfaces or interstitial surfaces is hydraulically the formation and growth of the boundary layer, it was attempted to investigate the probable deviation of the Manning resistance coefficient in non-uniform flows relative to their amount in uniform flows and the result of the research showed that the resistance coefficients in non-uniform flows were less than uniform flows and the necessity of correction of the resistance coefficients in culverts and dewatering channels.

Keywords: Boundary Layer, Resistance Coefficient, Manning Coefficient, Non-Uniform Flow

INTRODUCTION
The purpose of this study is to investigate the effect of boundary layer growth on the amount of passing discharge from box culverts and rectangular and trapezoidal channels and to calculate the process of variation of resistance coefficients against non-uniform flow and compare it with the coefficients provided for uniform flows.

One of the main factors causing non-uniform flow in the waterways is the formation and growth of the boundary layer in the vicinity of the solid surface. First of all, we should consider the boundary layer theory and relations presented in this field, and then we use the relations in studying the behavior of flow.

The presentation of the boundary layer theory by Prof. Prandtl in 1904 can be regarded as the beginning of the dramatic changes in the field of fluid mechanics. Since the actual fluid is viscous, a layer of this fluid adheres to the solid surface and, as a result of the collision of moving fluid particles with fluid particles sticking to the surface decrease their velocity, and, finally, these changes in velocity is adjacent to the surface of the velocity gradient in the direction of perpendicular to the axis of flow, this gradient starts from the adjacent solid surface at zero speed and continues to reach the
maximum velocity. Gradient height refers as the velocity of the thickness of the boundary layer. So, the thickness of the boundary layer is the distance from the solid surface to the point at which the fluid particles have maximum velocity. Within the boundary layer, shear stresses are tangible and cannot be ignored, it is clear that due to the formation of a velocity gradient in this layer, shear stresses are formed and the drag force acting on the surfaces also results from the formation of the boundary layer on them, it can be supposed ideal fluid with high precision outside of this layer. Inside the boundary layer, the flow can be laminar, interstitial, and turbulent, that the type of flow is a function of the point Reynolds, and depending on the flow of inside of the boundary layer, different relationships exist to estimate the thickness of this layer and the amount of stress, and consequently the amount of drag force.

**Figure 1(A, B). Boundary Layers formed on the floor of channel**

In the open channels, the stages of the formation of the boundary layer are as follows, and the following figures 1 indicate the boundary layers formed on the floor of channel from the dewatering site to downstream, and for the walls of the channel, the same boundary layers can be drawn that In the case of a wide channel, the boundary layers formed on the walls will have little effect on the velocity and passing discharge diagrams.

**Figure 2. The Relationship between the Reynolds Number**

Transition in the boundary layer from the laminar flow to the turbulent flow depends to many factors that most important of them are the point Reynolds number (in the investigated site), the amount of turbulent flow outside the boundary layer and the amount of roughness of solid surface. Of course, the phenomenon of the transition is an intermittent process and it begins with the occurrence of minor disturbances, and this continues until the flow is completely collapsed and the motion of particles gets irregular and random. The figure 2 below shows the relationship between the Reynolds number and that adjust by Hansen. It can be seen that the numbers are less than $3.2 \times 10^5$, the gradient of the graph is constant and from this point a sudden change in gradient is caused. To illustrate the effect of turbulent flow, we define the percentage of it. Percentage of being turbulent is the average time ratio of the amount of velocity component at a given point to the average time velocity at that point. The effect of the turbulent flow is also illustrated by subarea and Scrum Stats research’s. In the region between the two curves, the flow may be laminar or turbulent at any given moment. Due to the many factors involved in the transmission phenomenon, one can not specify a particular Reynolds number as the boundary of flow transformation from laminar to turbulent. The roughness of the surface causes the transition from a laminar flow into the turbulent, and even heating the environment or solid surface also accelerates the transition. However, the range of the Reynolds number in the transitional area varies in different sources and according to the most reliable sources, number 500,000 is known as the beginning of the transitional period or the end of the laminar region and the number 600,000 is known as the end of the transitional period or the beginning of the turbulent region.
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**Figure 3. The Relationship between the Reynolds Number and delta**

**Figure 4. The Relationship between the Reynolds Number and Turbulent Percentage**

If \( \text{Re} \leq 5 \times 10^5 \rightarrow \) the boundary layer is laminar

\[ \delta = \frac{5 \chi}{\text{Re}^0.5} \]

If \( \text{Re} \geq 6 \times 10^5 \rightarrow \) the boundary layer is turbulent

\[ \delta = \frac{0.37 \chi}{\text{Re}^{0.2}} \]

Often, the interstices are ignored because of their short length, but in long channels, the entire boundary layer can be considered as turbulent, because the length of the laminar area will be negligible against the length of the turbulent area.

In addition to the 3 main areas mentioned, we should refer to another region, and it is a laminar underlying layer formed under a turbulent boundary layer and the flow is laminar. The thickness of this layer is a criterion for determining the roughness and fineness of a surface hydraulically that the comparison criterion is as follows:

- **Smooth surface**
  \[ \text{if} \quad \delta' > 3 k_s \]

- **Interstitial surface**
  \[ \text{if} \quad 0.12 k_s \leq \delta' \leq 3 k_s \]

- **Rough surface**
  \[ \text{if} \quad \delta' < 0.12 k_s \]

In the above relations, \( k \), the height of roughness of surface and \( \delta \) the thickness of the lower layer is obtained from the following equation.

\[ \delta = \frac{1.16 \chi v}{U} \]

The velocity distribution in the turbulent area consists of 4 parts. The linear distribution of the velocity in the lower laminar layer, the uncertain distribution of the velocity in the buffer zone, the logarithmic distribution, and finally the exponential distribution, which accounts for 80% of the total thickness of boundary layer, therefore it can be approximately known that the distribution of velocity in the entire turbulent area is exponential and the general relation of velocity distribution in this region is as follows:

\[ \frac{v}{v_{max}} = \left( \frac{\delta'}{\delta} \right) \]

All relations presented in the context of the boundary layer are based on the assumption of non-slipping flow on the surface.

First, we study the limiting states of the flow, and then expand the discussion for the open channels, since a closed section can geometrically act both in open and closed manner, from a hydrodynamic point of view, it is clear that the relations governing the open flow behavior are the same relations that govern the closed flow behavior that takes a different form, and this change in form is due to the difference in existing conditions in two open and close modes.

The above limit modes are the boundary of the flow transform from close to open as described below.

We assume that the passing water level of culvert is free and is exposed to the atmospheric pressure, or, in other words, the culvert has an open function, and from now it is called mode 1.

The flow is approximately covered by the entire culvert section, and we know that the gradient and the maximum allowed velocity in ordinary culverts are negligible, and we can show the input velocity to the channel in the opening of
the input that has the uniform distribution with $V_{\text{max}}$, below figure represents the mode one:

We now consider a mode in which the flow completely fills the channel, but the upper surface of the water runs parallel to the upper edge of the culvert, or, in other words, the water does not hold on the upper lip, which makes it possible know pressure at passing flow levels is same as atmospheric pressure.

As seen in the figure of mode one, the boundary layer begins to grow from the beginning of the culvert (the floor surface of and the walls), and after passing through the laminar and interstitial areas, it enters the turbulent area and to the end of the channel this behavior of the flow is dominant. The turbulent boundary layer can grow to the surface of the water, so the maximum thickness of the boundary layer is equal to $a$, and the maximum thickness of the boundary layer formed on the walls is equal to $a/2$.

Because of the durability of the channel, the length of the laminar and transitional boundary layer against turbulent boundary layer is tangible and is usually neglected in the calculations of the laminar and transitional regions, and it is assumed that the turbulent boundary layer has been formed and grown from the very beginning of the culvert that by assuming that the computational drag force is slightly greater than the actual value and there are correlations to correct the Drag coefficient that is not mentioned.

In the mode two, the boundary layer is formed from the beginning of the culvert (the floor surface, the ceiling and the walls) and after passing the laminar and transitional regions reach to the turbulent area that extends to the end of the channel, in this mode the maximum thickness of The boundary layer formed on four faces equals to $a/2$. In the mode two, as in mode one, one of laminar or transitional regions are neglected due to their negligible length in front of the turbulent area, so the growth of the boundary layer begins from the beginning with the turbulence of the layer.

After the boundary layer reaches its maximum growth, after the length of $l_1$ in the first mode and after the length of $l_2$ in the second mode, the flow becomes uniform and the flow conditions are completely stabilized and no change in the flow behavior is observed and the velocity distribution curves is displayed in both modes.

The velocity distribution in both two modes is two-arc and undergoes changes in two directions, so if the direction of flow, $v$, $x$ is the velocity of the point in progress ($u=w=0$ other components are zero-speed), in this mode $dv/dz$ and $dv/dy$ is not zero.
As stated above, in the bulk of the thickness of the turbulent boundary layer, the velocity distribution is exponential, and it is generally assumed that the velocity distribution totally is exponential thickness, so the equation governing on the process of point velocity changes is as follows:

$$\frac{dV}{d\delta} = 0 \quad \frac{dV}{dx} = 0$$

In this equation:
- $Y$: distance from solid surface
- $\delta$: thickness of boundary layer in the given point
- $V$: point velocity
- $V_{\text{max}}$: maximum possible velocity

We know that the thickness of the turbulent boundary layer is, according to the recommendation of the Bellasius, obtained from the following relation:

$$\delta = \left( \frac{0.37 \cdot x}{Re_x \left( \frac{\delta}{\delta_x} \right)} \right)$$

In this equation:
- $\delta$: thickness of turbulent boundary layer
- $x$: distance of given location from the beginning of the channel
- $Re_x$: point Reynolds number

Now, lengths of the $l_2$, $l_1$ that were previously introduced are simply computable, and at the ordinary temperature for water the relations for water present on the next page.

The following relationships can be used to find $l_1$, $l_2$, in which these relations there are two unknown $l_1$ (or $l_2$), and an in view of the exponentially of the above equations, it is not easy to determine relations in terms of $l_1$ (or $l_2$) and $a$, but if we consider the degree of significance of the terms of the above relations or the sensitivity of the above equations by changing their variables, we find that the term $V_{\text{max}}$ versus $l_1$ (or $l_2$) and $a$, is less important.

$$a = \frac{0.37 \cdot l_1}{\left( Re_x \right)^{\frac{1}{2}}} = \frac{0.37 \cdot l_2}{\left( Re_x \right)^{\frac{1}{2}}} \Rightarrow \frac{0.023}{V_{\text{max}}} \frac{l_1^{\frac{1}{2}}}{l_2^{\frac{1}{2}}}$$

$$l_1^{\frac{1}{2}} = \frac{a \cdot V_{\text{max}}^{\frac{1}{2}}}{0.023} \Rightarrow l_1 = 11.64 \cdot a^{\frac{1}{4}} \cdot V_{\text{max}}^{\frac{1}{4}}$$

$$a = \frac{0.37 \cdot l_1}{\left( Re_x \right)^{\frac{1}{2}}} = \frac{0.37 \cdot l_2}{\left( Re_x \right)^{\frac{1}{2}}} \Rightarrow \frac{0.023}{V_{\text{max}}} \frac{l_1^{\frac{1}{2}}}{l_2^{\frac{1}{2}}}$$

$$l_2^{\frac{1}{2}} = \frac{a \cdot V_{\text{max}}^{\frac{1}{2}}}{2 \cdot 0.023} \Rightarrow l_2 = 46.94 \cdot a^{\frac{1}{4}} \cdot V_{\text{max}}^{\frac{1}{4}}$$

We know that the maximum velocity allowed in ordinary culverts is from 1.5 to 2.5 m/s, due to the fact that they are prevented from scouring down the culvert and sometimes there are culverts with a high slope and velocity that at the bottom of them De-energizing structures are used, which are not at all reasonable and cost-effective.

Given the fact that in discharge, there is control plane for the maximum flow velocity, we can consider the expression $V_{\text{max}}$ equal to one in the range of the above velocity. To be more applicable, the existing coefficient relationships are moderated to some extent, and eventually we will have:

$$l_1 = 133 \cdot a^{\frac{5}{4}}$$

$$l_2 = 56 \cdot a^{\frac{5}{4}}$$

The above relations represent the exponential relationship of $l_1$ (or $l_2$) and $a$, and with the presence of $a$, we can obtain the numerical value of $l_2$, $l_1$. In addition, the above equations can be used in addition to the box culverts for pipe culverts (a=D) and rectangular and trapezoidal channels.

If we divide the obtained relations, the ratio of $l_1$ to $l_2$ will be greater than 2, which is due to the exponential of the initial relations and may seem a bit far from the logic, which indicates a low growth rate of the boundary layer in a mode 1 to mode 2, so that the boundary layer formed in mode 1 is more smooth and with less slope than the base level, and therefore the thickness of displacement that will calculate in the future
will have a lower growth rate (lower slope) than mode 2.

\[ \frac{L_1}{L_2} = 2.38 \]

From the above ratio, it can be concluded that the resistance of the channel one to the flow is much less than that of the second channel. We know that moving fluid particles on a straight line can only have tangential acceleration. Now, we imagine that consuming the energy (which is generated by the boundary layer) will bend the previous flow line, in the current situation, fluid particles are affected by inertia in addition to tangential acceleration, centric acceleration or normal acceleration over the flow line, although the new tangential acceleration is not necessarily equal to the previous tangential acceleration. Therefore, the vertical acceleration and its amount are affected by the resistance to flow and the presence of vertical acceleration can be considered as the main factor in the change of pressure. But if we compare the above-mentioned conditions with modes 1 and 2, then we find that, according to the definition of the limiting state, the distribution of pressure is hydrostatic in both modes, so the above conditions appear in another way, and it is increasing in overall losses and consequently dropping the passage discharge.

Here we briefly discuss the thickness of the displacement and the resulting relations.

If the passing fluid is through the culvert of ideal fluid, and we want the mass discharge passing through the culvert in these conditions would be equal to the state of the actual fluid flow in the culvert, then the culvert cross section could be reduced and the displacement level of each of the surfaces (Vertical and horizontal surfaces) to the initial location of that surface, is called thickness of the displacement, and display it with \( \delta_\ast \), so this thickness is not physically existence and is completely hypothetical, and its mathematical interpretation is as follows.

\[
\delta_\ast = \int_0^\delta \left(1 - \frac{V}{V_{\max}}\right) dy
\]

\( V \) expresses the point velocity from the solid surface to height \( \delta \), and \( dy \) represents the integral direction in the height of the culvert, and is, as a rule, converted to the \( dz \) for vertical surfaces.

In the laminar boundary layer, the proposed velocity distribution is usually used by Prenatal:

\[
\frac{V}{V_{\max}} = \frac{3}{2} \left(\frac{y}{\delta}\right) - \frac{1}{2} \left(\frac{y}{\delta}\right)^3
\]

And in the turbulent boundary layer, the exponential velocity distribution that mentioned before, has been used, which, if inserted in the equation \( \delta_\ast \), eventually results in the amount of displacement thickness for a laminar and turbulent boundary layer.

laminar boundary layer

\[ \delta_\ast = 0.375\delta \]

Turbulent boundary layer

\[ \delta_\ast = 0.125\delta \]

\[ \delta_\ast = 0.0025 L \frac{V}{\delta} \]

This relation was obtained with previous assumptions and given the low importance of velocity term (at low velocity), where \( L \) is the initial distance of the channel to the point under consideration (here it is the total length of the culvert).

The passing discharge has a direct linear relationship with the cross-sectional area of the channel. According to the concept of displacement thickness, we can find the effective cross-section in the first and second modes, which we have for mode one:
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$$L < l_1 \Rightarrow A_i = a^2 - 0.0075 a L 0.5$$

$$l_i < L < l_1 \Rightarrow A_i = 0.875 a^2 - 0.0025 a L 0.5$$

$$L > l_1 \Rightarrow A_i = 0.75 a^2$$

And in mode 2, effective cross-section is:

$$L < l_2 \Rightarrow A_{ii} = a^2 - 0.01 a L 0.5$$

$$L > l_2 \Rightarrow A_{ii} = 0.75 a^2$$

As we can see, the above equations are simple functions of an and L, and with the involvement an and L, an effective cross section A I and A II can be calculated and compared with a 2 that is the total cross section. Finally, the actual discharge is obtained from the product of the maximum discharge in the ratio of effective cross-sectional area to the total cross-sectional area.

$$Q_{ref} = \left(a^2 \times V_{max}\right) \left(\frac{A_1}{a^2} or \frac{A_{ii}}{a^2} or \frac{A_{III}}{a^2}\right)$$

In order to extend the subject to other probable conditions, we present relationships as A III that are true for y<a (y flow depths to the floor of channel) and A I is also a special case of general relationships of A III. Of course, there are six distinct relationships for A III that here we can refer to them as two main equations:

$$L < l_1 , L < l_1 \Rightarrow A_{III} = a \cdot y - 0.0025 a L 0.5 - 0.005 y L 0.5$$

$$L > l_1 , L > l_1 \Rightarrow A_{III} = a \cdot y - 0.0025 a l_i 0.5 - 0.005 y l_i 0.5$$

Four other relationships can also be used for interstitial modes and when \( l_1 > l_2 \) and or \( l_2 > l_1 \). With respect to A I and A III, the graphs are given which represent the trend of changes in the effective surface of the flow relative to the changes in the growth of the boundary layer along the path, and finally, the drawn curves are analyzed.

According to the drawn graphs, it can be concluded that the process of variation of the flow resistance coefficients against flow follow of such graphs. In other words, the amount of these coefficients assumed to be constant in a uniform and non-uniform flow is changed in a non-uniform flow mode and is not fixed, typically, for these coefficients, the table is based on the material of the channels, which is used for uniform and non-uniform flows. In recent years, there have been numerous measures to calculate the trend of these numbers variations for non-uniform flow, and we know that discharge calculate from flow Sheri or Manning relations strongly depend on the amount of resistance coefficients and the exact determination of the numerical value is very important, according to previous graphs we can mention following equation for y<a:

$$n = \left(1 - \frac{\delta}{8y}\right) \frac{R_h 0.5 \cdot S_f 0.5}{(1 - k) \cdot V}$$

\(\delta\): thickness of boundary layer at given point

V: average velocity of flow at given section

K: the ratio of effective cross-section to total cross-section

Si: frictional slope

Rk: hydraulic radius

Figure 8. The Flow Resistance Coefficients against flow

Figure 9. Ratio in Non-Uniform Flows

CONCLUSION AND RESULT

The ratio \( (1 - \frac{\delta}{8y}) \) is 1 in uniform flow and is constant. If the value of this ratio in non-uniform flows varies depending on the non-uniformity of the flow between 0.88 and 1, then we conclude that n value in the non-uniform
flow is less than that of the uniform mode and the rate of this decrease could be up to 12%. Of course, a careful examination of the process of changes requires extensive testing. Given the existing empirical relationships for converting the Manning coefficient to the Sheri coefficient or the Darsi-Vailsbakh coefficient, we can also study the changes in these two coefficients.

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