A Note on Visualizing the Optimal Time for Closing a Momentum Trade

Reza Habibi

Iran Banking Institute, Central Bank of Iran

*Corresponding Author: Reza Habibi, Iran Banking Institute, Central Bank of Iran

ABSTRACT

In this paper, a simple method is presented to visualize the optimal time for closing a momentum trade. The decision criterion is based on discounted financial asset price (like stock) at which discounted price is computed with a minimum attractive rate of return. It is seen that as soon as, the discounted price starts decreasing is the best time for closing a momentum trade.

Keywords: Geometric Brownian motion; Momentum trade; Stopping time

INTRODUCTION

Ekstrom and Lindberg (2011) proposed a strategy, based on Bayesian posterior probability, for optimal closing time of a momentum trade. They supposed that the financial asset price (say a stock) obeys a geometric Brownian motion with a change point in drift. Indeed, they assumed that

\[ ds = \mu_s dt + \sigma dB_s, \]

where \( B_s \) is a Brownian motion and \( \mu_t = \theta_1 \) for \( t \leq \tau \) and \( \mu_t = \theta_2 \) for \( t > \tau \). The optimal strategy solution is obtained by

\[ V = \max_{\tau \in \mathcal{F}} E(e^{-\tau \nu} s_{t^*}), \]

where \( \mathcal{F} \) is the collection of all stopping times \( \tau^* \). However, one can see that

\[ E(e^{-\tau \nu} s_t) = \begin{cases} (\theta_1 - \nu) \tau & t \leq \tau \\ (\theta_1 - \theta_2) \tau - (r - \theta_2) t & t > \tau. \end{cases} \]

This function attains its maximum at \( \tau^* \) if \( \theta_2 < r < \theta_1 \). If \( \tau \approx \infty \) with probability one, then \( E(e^{-\tau \nu} s_{t^*}) = E(e^{-\tau \nu} s_t) \), using the optional sampling theorem. Indeed, for all \( \tau^* < \infty \), then \( E(e^{-\tau \nu} s_{t^*}) \) attains its maximum at \( \tau \). Here, conditions are extracted to make sure that

\[ M_t = \frac{e^{-\tau \nu} s_t}{E(e^{-\tau \nu} s_t)}, \]

is close to 1 and then \( e^{-\tau \nu} s_t \) is used instead of \( E(e^{-\tau \nu} s_t) \). It is easy to see that

\[ M_t = \exp \left( \sigma B - \frac{\sigma t}{2} \right) = \exp \left( \sigma (B - \frac{t}{2}) \right), \]

and that \( M_t \) is a martingale with respect to filtration \( \sigma(B_u, u \leq t) \), i.e., the sigma-field generated by \((B_u, u \leq t)\). It is clear that when \( \sigma \to 0 \), then \( M_t \to 1 \). Also, Doob inequality (see Bjork, 2009) implies that, for some maturity \( L \), then

\[ P(\sup_{0 \leq \tau \leq L} M_t > \epsilon) \leq \frac{E(M^\alpha)}{\epsilon^\alpha} \]

for some \( 0 < \alpha < 1 \). Also, \( E(M^\alpha) = \frac{-\alpha(1-\alpha)}{2} L \). Thus,

\[ E(M^\alpha) = \exp \left( \alpha \left( \frac{1}{2} \right) \right) \leq \exp \left( \frac{-\alpha}{2} - L + \log(\epsilon) \right). \]

Assuming \( \frac{\sigma^2}{L} = -\log(\epsilon) \), then \( L = \frac{-2\log(\epsilon)}{\sigma^2} \).

Thus, by sequentially search for existence of momentum during intervals with length \( L \), recursively, as soon as \( e^{-\tau \nu} s_t \) starts to decrease, that time point is a suitable point for selling the asset. The following proposition summarizes the above discussion.

Proposition 1

Assuming \( L = \frac{-2\log(\epsilon)}{\sigma^2} \), by search recursively time intervals with length \( L \), the momentum is detected.

Proof. It is discussed in section 1.

Thus, the estimate for \( \tau \) is the time point at which \( e^{-\tau \nu} s_t \) attains its maximum. That is,

\[ \hat{\tau} = \arg\max_{\tau} \{e^{-\tau \nu} s_t \}. \]

Since, \( e^{-\tau \nu} s_t \) is too close to its mean and \( E(e^{-\tau \nu} s_t) \) takes its maximum at actual change
point $\tau$, thus, $\hat{\tau}$ is a consistent estimator for $\tau$. The following proposition summarizes this fact.

**Proposition 2**

The estimator $\hat{\tau}$ is a consistent estimator for $\tau$.

The rest of the paper is organized as follows. First, the simulation results are derived in the next section. Section 3 concludes.

**SIMULATIONS**

Here, using the Model Risk adds-in of Excel, this case is simulated that $\theta_1 = 0.002, \theta_2 = 0.007, \tau = 543$, initial value of stock is 0.38 $, \sigma = 0.025$ and $\varepsilon = 0.01$. Let $\tau = 0.005$. The time period of study is 1000 days. The following plot shows the time series of $e^{-\tau t} s_t$ which implies that there is a change about 543.

![Figure 1. Time series plot of $e^{-\tau t} s_t$](image1)

While the time series of $s_t$ is given as follows. However, the Fig.1. has better visual interpretation.

![Figure 2. Time series plot of $s_t$](image2)

As follows, the empirical distribution of $\hat{\tau}$ is given. Clearly, $\hat{\tau}$ is concentrated well on actual momentum time $\tau$.

![Figure 3. Histogram of $\hat{\tau}$](image3)

Here, a threshold $K$ is given such that $P(\max_t e^{-\tau t} s_t \leq K) = 1 - \alpha$. The following table gives some values for $K$, for various selections for $\alpha$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.1</th>
<th>0.05</th>
<th>0.025</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>2.71</td>
<td>3.42</td>
<td>3.79</td>
<td>4.81</td>
</tr>
</tbody>
</table>

**CONCLUSION**

The argmax time point of $e^{-\tau t} s_t$ is a consistent estimator of actual momentum point. The time series plot of $e^{-\tau t} s_t$ has a good visualization for momentum time point.

**REFERENCES**
